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Application of Fractals in the Acoustic Emission Study of Micro-cracking and Fracture Development in Brittle Rock

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Abstract

Acoustic emission (AE) signatures of the various stages of micro cracking activity leading to fracture development in brittle rock have been subjected to fractal analysis. The AE monitoring experiments were carried out under controlled stress conditions in the laboratory. The results show that the size distribution of micro cracks follows a simple 'power law'. Whereas the spatial and temporal distribution, and clustering properties of micro cracks and fracture development in rock, or for that matter in any quasi-brittle material, is quite complex. It can however be investigated using the 'correlation dimension method' of fractal geometry. In this paper, both the fractal methods are briefly outlined and the results obtained from the experiments carried out on few earthquake-prone rocks are presented and discussed.

Key words: a*coustic emission,b-values, hypocenters, power law exponent, fractal dimension*

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1. Introduction

Fractal is a new concept that has caught the attention of many researchers in recent years after it was introduced by Mandelbrot[1] in a geological context. It is used to describe and quantify many uneven and non-smooth objects, irregular events, phenomena and process in nature, as well as those resulting from Industrial engineering and production. Mathematically, a fractal is defined as an object whose fractal dimension (which will be defined latter)is greater than its topological dimension obtained by the usual Euclidean concepts of length, area etc. Fractal is a scale-invariant structure possessing the property of 'self-similarity'. The self-similarity means that any small portion of a fractal, when magnified by an arbitrary factor, looks the same as the original fractal which in turn is similar to the whole object[(2.3).].In other words it is 'scale-invariant'. All fractals are restricted to a specific range of scales for which scale-invariance applies, and it is important to specify the upper and lower limits as well as the fractal dimension (D) which applieswithin that range[1, 2]. Whether one looks around in the nature at very large scales (coastlines, mountain ranges, landscapes, earthquakes, rivers, forests etc) or at the microscopic andatomic level or even at human physiology and blood vessels, fractals seem to exit everywhere. In the present context, the micro crack/ crack populations leading to rock fracture at several scales from laboratory samples to earthquakes are all fractals [4-11]

The simplest and most straight forward methods for monitoring the micro cracking activity and fracture behavior of rock under stress is to count and record the number of AE events and analyse their statistical behavior. Among the signal parameters, the amplitude distribution analysis of AE population and various subsets of it have provided better insights to analyse the various stages of rock fracture in terms of the frequency-magnitude relationship (*b*-value) of AE following the methods adopted in seismology[12, 13]. Furthermore, the technology to locate the sources of AE has vastly improved and made the spatial-temporal distribution analysis of micro crack damage in the volume of rock also possible in terms of AE hypocenter data[14–19]. Furthermore, in view of the advantages of fractal geometry, the application of fractals has gained special significance in order to give a better description and also quantify the size distribution as well as the clustering properties of micro cracks in rocks undergoing fracture [4, 6,10,11,20]. In all such endeavours, the "power law exponent method" was used to determine the fractal dimension, D of the size distribution, and the "correlation dimension method" was used to determine the fractal dimension, D_C of the spatial-temporal distribution of micro crack damage in rocks subjected to fracture under controlled laboratory conditions as discussed in detail elsewhere [20]. The fractal measurement methods for the determination of fractal dimensions are as follows. However, the fractal dimension obtained by different methods should not be compared too literally with each other because they reflect different aspects of the scale invariance. For example, the 'power-law exponent (D)' measures the relative proportion of large and small seismogenic faults or micro cracks / cracks producing AE[21]; the 'correlation dimension (D_C)' is a measure of the spacing or clustering properties of a set of points representing either earthquake or rock burst epicenter distributions[4,8,9], or hypocenter distributions of AE[5,6,11], and the 'capacity dimension (D_0)' measures the space filling properties of a fractal set with respect to changes in grid scale[3].

2. Fractal measurements

2.1 Fractal dimension of size distribution (Power-law exponent): The number of AE events generated during the laboratory compression tests on rocks is approximately proportional to the number of newly formed micro cracks, and AE amplitudes that are usually measured in dB are proportional to the size of the crack or crack growth increments (7 Main et al 1993). Therefore, the best way to examine the magnitude (amplitude / 20) distribution of AE is to plot the number (N) versus magnitude (M) plots of AE in a logarithmic scale and estimate the *b*-value (slope of the negative gradient of the power-law) using either the well known Gutenberg-Richter relationship or Aki's equation that are used for the analysis of earthquakes as well as the AE occurring at the laboratory scale for all materials [7, 12]. The equations are as follows:

GBR : $\text{Log } N = a - bM$ .... (1)

AKI: $b = \text{Log}_{10}e / (M - M_0)$ .... (2)

Where 'a' is a constant, M_0 is the threshold magnitude (or amplitude/ 20) and 'b' is the slope of the straight line portion of the log linear frequency-magnitude distribution plot. The precise values of a and b (scaling constants) are pendent on the rock type and the loading conditions. Generally b is in the interval $0.5 < b < 2.5$. A high AE *b*-value arises due to a number of relatively small AE events representing new crack formation and slow crack growth, whereas a low *b*-value indicates faster or unstable crack growth accompanied by relatively high amplitude AE in large numbers. The fractal dimension of the size distribution, D is related to *b*-value as follows[7]:

$$D = 3b / c \quad .... \quad .... \quad .... \quad .... \quad .... \quad (3).$$

In general, c = 3/2. Thus the above equation can be rewritten as

$$D = 2b \quad .... \quad .... \quad .... \quad .... \quad (4).$$

2.2 Fractal dimension of spatial distribution (Number - radius relationship): The AE or micro seismic or seismic event locations construct a spatial distribution of a point set in which a point corresponds to a cracking surface or volume element in physical space. Thus the fractal dimension of the damage evolution process at any given scale can be directly measured from the distribution of the point set[8].Considering a sphere with radius r, the total number of events inside this sphere over the distribution can be counted and denoted by M(r). A set of data $M(r_i)$ associated with different radii r_i (i = 1,2,3, ...) can be obtained from fractal geometry. There is a relation between $M(r_i)$ and r_i in the form $M(r) = r^1$ for the line distribution of point set, $M(r) = r^2$ for the plane distribution, and $M(r) = r^3$ for the 3-D (or volume) distribution, and

$$M(r) = r^{Dc} \quad .... \quad .... \quad .... \quad .... \quad (5),$$

for a fractal distribution. The above equation is also called the number - radius relation and the fractal dimension, D_c is called the clustering dimension which is equal to the slope of the log M(r) - log(r) plot. In this fractal measurement, the center point of the spheres with different radii r_i is chosen as the mass center of the distribution.

***2.3Fractal dimension of spatial distribution (*Correlation exponent):**

Spatial self-similarity can be demonstrated by examining thedistribution of distances between pairs of points in a data setover a range of distances. This has been done on the earthquake scale[5]and on the laboratory acoustic emission scale[4]$_4$using a spatial two-point correlation function. It is given as follows

$$C(r) = [2N_r (R < r) / n(n-1)]. \quad .... \quad .... \quad .... (6),$$

Where Nr(R < r) is the number of event hypocenter or epicenter pairs with a distance smaller than r, and n is the total number of events. If the distribution of hypocenters or epicenters has self- similar structure, C(r) can be expressed in the form

$$C(r) = r^{D} \quad .... \quad .... \quad .... \quad .... \quad (7),$$

where D is a kind of fractal dimension called the correlationexponent that gives the lower limit of the Hausdorff dimensions. This method was adopted by us for investigating the fractal character of the AE hypocenter distributions of rocks at the laboratory scale [11, 19]

3. Laboratory experiments

The rocks tested include some hornblende schists and amphibolites of NX size (50 mm dia and 100 mm long drill cores) from the deep mines of Kolar gold fields; and basalts, granites and migmatite gneisses of BX size (30 mm dia and 60 mm long drill cores) from the basement of Deccan Volcanic Province (DVP) which has been experiencing prolific seismicity. The rock samples of KGF were tested in Japan under incremental and creep loading conditions at constant confining pressure of 30 MPa using a multi-channel AE monitoring and source location system (Satoh et al 1996, Lei et al 2000). The tests on basement rock samplesof DVP were carried out under uniaxial and triaxial compression using a 2-ch PAC AE monitoring unit and software (Mistras) for processing and analysing the AE statistics and signal parameters[20] .The methods adopted for fractal analysis of the stress-induced damage in rock are described in detail in our earlier papers [10,11,20].

4. Results &Discussion

4.1 Fractal analysis of size distribution of micro crack populations in rock:

The AE *b*-value data obtained during some of the controlled laboratory tests performed under trivial compression and creep has been very useful to investigate the fault nucleation and its quasi-static fault growth in jointed amphibolite rocks[10,11]intact hornblende schists[18,19]of the Kolar gold mines; and Latur basalts and granitic rocks of different grain size of Koyna basement[20]. Further the mechanics of brittle deformation and crack growth could be inferred from AE statistics because the number of AE events is proportional to the number of growing cracks, and AE amplitudes are proportional to the length of crack growth increments in rock[21]. The AE *b*-value (or D) data obtained during the uniaxial compression testsof granites and the basement rocks of DVP show that *b* is ~ 1.0 (i.e., D ~ 2) during a large portion of the loading regime. As the impending failure approaches in the rock, during the nucleation phase of the micro crack damage and unstable crack propagation, the AE *b*-value not only decreases sharply to as low as 0.5 (or D = 1.0) for hard rocks but also shows short-term anomalies in terms of the underlying physical processes of crack growth in rocks containing weak planes and grain size anomalies[10.11.20].

4.2 Fractal analysis of spatial distribution of micro crack damage in Kolaramphibolites: The hypocenter data of 1800 AE events recorded during the triaxial compression and creep tests at 30 MPa confining pressure in the GSJ laboratory on a jointed amphibolite rock sample and few hornblende schistsof the Nundydroog mine, KGF were processed and analysed[(11,18.19).]. The D_C values computed from the slopes of log C (r) versus log (r) during the primary, secondary and tertiary stages of creep of the jointed amphibolite were found to be 0.67, 1.07 and 1.82 respectively. These observations indicate that the microcracks which concentrated more on the joint plane during the incremental loading and primary creep weakened the material resulting in low D_c of 0.67. Subsequently, the micro cracking activity during secondary and tertiary creep regimes shifted on to the eventual fracture plane with diffused AE activity. These observations are quite useful for the interpretation of seismic activity associated with fault zones in rock masses.

5. Conclusions

1. The fractal character of micro cracking and fracture development has been investigated successfully in rocks at the laboratory scale.

2. Fractal analysis of AE data accompanying rock fracture at the laboratory scale can yield a better description and quantification of the size and spatial distribution of damage evolution in terms of fractal dimension in intact as well as jointed rocks under a variety of loading conditions.

3. The size distribution of microcracking in several rocks was investigated using the ‘power-law exponent method’ and AE *b*-value data, the spatial and temporal distribution of micro cracking and fracture development in the burst-prone rocks of Kolar gold mines were analysed with the help of AE hypocenter data and the ‘correlation dimension method’ of the fractal geometry.

4 The state of criticality of rock under stress can be more accurately identified and tracked in terms of fractal dimension for a better prognosis of rock failure and to predict and control catastrophic rock failures.

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References

1. Mandelbrot BB (1982): ***The Fractal Geometry of Nature*.** Freeman Publ., San Francisco.

2. Turcotte DL (1986): Fractals and fragmentation. *J. Geophys. Res.,* 91, 1921-1926.

3. Feder T (1987): ***Fractals.***Plenum Publ., London.

4. Hirata T, Satoh T, and Ito K (1987): Fractal structure of spatial distribution of microfracturing in rock. *Geophy. J., R. Astron. Soc.* 90, 369 - 374.

5. Hirata T (1989): A correlation between the *b*-value and the fractal dimension of earthquakes. *J. Geophys. Res.,* 94, 7507-7514.

6. Lei XL, Nishizawa O, Kusunose K, and Satoh T (1992): Fractal structure of the hypocenter distribution and focal mechanism solutions of AE in two granites of different grain size. *J. Phys. Earth,* 40, 617-634.

7. Main IG, Sammonds PR, and Meredith PG (1993): Application of a modified Griffith criterion to the evolution of fractal damage during compressional rock failure. *Geophys. J. Int.,* 115, 367-380.

8. Xie H, and Pariseau WG (1993): Fractal character and mechanism of rockbursts.*Int. J. Rock Mech. Min. Sci., &Geomech. Abstr.,* 30, 343-350.

9. Shivakumar K, Rao MVMS, Srinivasan C, and Kusunose K (1996): Multifractal analysis of the spatial distribution of area rock bursts at Kolar gold mines. *Int. J. Rock Mech. Min. Sci., &Geomech. Abstr.,*33, 167-172.

10. Rao MVMS, Shivakumar K, Kusunose K, and Lei XL (1999): Fractal analysis of acoustic emission and its application in the investigation of compressive fracture of brittle rock. *J. Nondestr. Test. &Eval.,* 19 (3), 60-67.

11. Shivakumar K, and Rao MVMS (2000): Application of fractals in the study of rock fracture and rockburst - associated seismicity. , Chapter 15, In *Application of Fractals in Earth Sciences.* (Ed.) VP Dimri, Balkema Publ., Amsterdam, pp. 171-188.

12. Rao MVMS (1996): Significance of AE based b-value in the study of progressive failure of brittle rock: Some examples from recent experiments. *Proc. 14th WCNDT, New Delhi, Dec. 1996,* (Eds.) CG Krishnadasnair et al, Oxford & IBH Publ., New Delhi, Vol. 4, pp. 2463-2467.

13. Rao MVMS and Prasanna Lakshmi KJ (2005): Analysis of b-value and improved b-value of acoustic emissions accompanying rock fracture. *Current Science,* 89 (9), 1577-1582.

14.Lockner D (1993): The role of acoustic emission in the study of rock fracture. *Int. J. Rock Mech. Min. Sci.&Geomech. Abstr.*30, 883-899.

15. Rao MVMS and Kusunose K (1995): Failure zone development in Andesite as observed from acoustic emission locations and velocity changes. *Phys. Earth Planet. Interiors,* 88, 131-143.

16. Satoh T., Shivakumar K, Nishizawa O, and Kusunose K (1996): Precursory localization and development of micro fractures along the ultimate fracture plane in amphibolites under triaxial creep. *Geophy. Res. Lett.,* 23(8), 865-868.

17. Lei XL, Satoh T, Nishizawa O, Masuda K, and Kusunose K (1998): A real time AE hypocenter monitoring system for laboratory rock fracture experiments. *Bull. Geol. Survey of Japan,* 49, 353-363.

18. Lei XL, Kusunose K, Rao MVMS, Nishizawa O, and Satoh T (2000): Quasistatic fault growth and cracking in heterogeneous brittle rock under triaxialcompression using acoustic emission monitoring. *J. Geophys. Res.,* 105, 6127-6139.

18. Lei XL, Satoh T, Nishizawa O, Kusunose K, and Rao MVMS (2005): Modelling damage creation in stressed brittle rocks by means of acoustic emission. In *Controlling seismic Risk.* (Proc. 6^{th} Intl. Symp. On Rockburst and Seismicity in Mines), Perth, Australia, (Eds.), Y. Potvin and M. Hudyma., pp. 327-334.

20. Rao MVMS (2012): ***Acoustic Emission Signatures of Microcrack damage in Rock: Laboratory Investigations.*** Lambert Academic Publishers, Germany, pp.1-153.

21. Main Ian G (1993): Damage mechanics with long-range interactions: correlation between seismic b-value and the fractal two-point correlation dimension. *Geophys. J. Int.,* 111, 531-544.